A Shape-based Quality Evaluation and Reconstruction Method for Electrical Impedance Tomography

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Abstract. Linear methods of reconstruction play an important role in medical Electrical Impedance Tomography and a wide variety of algorithms based on several assumptions exists. With the “Graz consensus Reconstruction algorithm for EIT” (GREIT), a novel linear reconstruction algorithm as well as a standardized framework for evaluating and comparing methods of reconstruction were introduced that found widespread acceptance in the community.

In this paper, we propose a two-sided extension of this concept by first introducing a novel method of evaluation. Instead of being based on point-shaped resistivity distributions, we use 2759 pairs of real lung shapes for evaluation that were automatically segmented from human CT data. Necessarily, the figures of merit defined in GREIT were adjusted. Second, a linear method of reconstruction that uses orthonormal eigenimages as training data and a tunable desired point spread function are proposed.

Using our novel method of evaluation, this approach is compared to the classical point-shaped approach. Results show that most figures of merit improve with the use of eigenimages as training data. Moreover, the possibility to tune the reconstruction by modifying the desired point spread function is shown. Finally, the reconstruction of real EIT data shows that higher contrasts and fewer artifacts can be achieved in ventilation- and perfusion-related images.

1. Introduction
Electrical Impedance Tomography (EIT) (Brown, 2003) seeks to estimate the electrical resistivity distribution inside a body from measurements on its surface. Several medical applications exist, ranging from breast and brain imaging (Bayford, 2006) over pulmonary monitoring (Frerichs, 2000; Leonhardt and Lachmann, 2012; Adler et al., 2012) to bladder filling level determination (Leonhardt et al., 2011). EIT also has applications in process control and geological science (Beck and Williams, 1996;
Samouelian et al., 2005). However, while the basic principle is the same, requirements
in terms of safety demands, temporal resolution and model accuracy might be completely
different.

As a non-invasive imaging technology, EIT has no known side effects like, for
example, x-ray. Especially in pulmonary monitoring applications, the use of electrode
belts with 16 electrodes has found widespread application and enables this technology
to be used for bedside monitoring. Additionally, other scenarios that use, for example,
minimally invasive electrode configurations, are currently under investigation (Nasehi
Tehrani et al., 2012; Czaplik et al., 2013). Recent experimental hardware is reported
to operate with up to 100 frames per second (Oh et al., 2011), but even commercially
available, medical approved devices allow a temporal resolution of up to 50 frames per
second. This makes EIT an ideal monitoring modality for fast physiological processes
that involve a change in resistivity, such as the pumping of blood or breathing.

2. Review of Inverse Solution Algorithms

There are two major mathematical problems in EIT, the forward and the inverse problem
(Lionheart et al., 2004). The forward problem seeks to calculate the vector of voltage
measurements $u$ on the body surface when the underlying resistivity distribution $Z$ is
known, i.e.

$$u = F(Z).$$

In an actual measurement scenario, this problem is solved by the laws of
electromagnetism. In a computational scenario, $u = [u_1, u_2, ..., u_M]^T$ can be obtained
either by analytical methods (if $Z$ is sufficiently simple, e.g. a circular resistivity
step in the center of a homogeneous, circular resistivity distribution) or by the Finite
Element Method (FEM) for arbitrary resistivity distributions. While it is challenging
to exactly model, for example, the influence of electrodes or anisotropic materials, the
basic problem is well-posed, i.e. a unique, stable solution is guaranteed to exist.

The inverse problem (Lionheart et al., 2004; Lionheart, 2004), on the other hand,
seeks to reconstruct the underlying resistivity distribution from measurements on the
boundary,

$$Z = G(u).$$

Unfortunately, here the properties of existence, uniqueness and stability of a solution
are in general violated, which makes this problem ill-posed. Consequently, a simple
inversion of the forward problem $G \neq F^{-1}$ is not possible in the general case. To
overcome this, some sort of a-priori knowledge has to be incorporated into the solution
of the inverse problem. A variety of algorithms exist and a lot of research is currently
conducted. Recent non-linear reconstruction algorithms are based on, for example, level-
set (Chung et al., 2005; Rahmati et al., 2012), D-bar (Isaacson et al., 2004) or $l_1$-norm
methods (Borsic and Adler, 2012). For clinical purposes, however, linear reconstruction algorithms still play a very important role due to their guaranteed stability and low computational costs. Early linear reconstruction algorithms used assumptions borrowed from the Computed Tomography (CT) reconstruction methods and were thus termed “back projection algorithms”, like their CT counterparts (Barber et al., 1983). Although based on oversimplified assumptions, this algorithm was widely used in the medical community and is listed for completeness. Other methods are based on the inversion of the linearized forward problem incorporating various forms of regularization. These are assumptions about an expected resistivity distribution like smoothness of the solution (Laplace Prior) or homogeneous (Tikhonov) or inhomogeneous (NOSER) (Cheney et al., 1990) penalties on large resistivity values.

Several of these methods as well as solvers of the forward problem are included in the “Electrical Impedance Tomography and Diffuse Optical Tomography Reconstruction Software” (EIDORS) (Vauhkonen et al., 2001; Polydorides and Lionheart, 2002; Adler and Lionheart, 2005), a community-based, open-source software package with the goal to “provide free software algorithms for forward and inverse modelling for Electrical Impedance Tomography (EIT) and Diffusion based Optical Tomography, in medical and industrial settings, and to share data and promote collaboration between groups working these fields.” (EIDORS: Electrical Impedance Tomography and Diffuse Optical Tomography Reconstruction Software, n.d.).

As a consequence, a variety of reconstruction methods exist that make their objective comparison (and therefore the evaluation of reconstructed EIT data) a challenging task. To overcome this problem, the “Graz consensus Reconstruction algorithm for EIT” (GREIT) was developed (Adler et al., 2009), which has two aspects to it: First, it consists of a method to evaluate arbitrary methods of reconstruction. For this, a set of circular resistivity distributions with a homogeneous background and small point-shaped resistivity disturbances of unit value are simulated. Since the quality of the reconstruction is expected to be cylindrical symmetric, every resistivity distribution is sufficiently described by the distance of the disturbance from the center. For each, the forward problem is solved via FEM. The resulting voltage vectors are then passed to the algorithm under test which solves the inverse problem and the result is examined using several figures of merit:

**Amplitude Response:** How large is the maximum resistivity value of the reconstructed distribution?

**Position Error:** How far is the center of gravity of the reconstructed resistivity distribution shifted relative to the ground truth?

**Resolution:** How large is the reconstructed distribution?

**Shape Deformation:** How much does the reconstructed distribution resemble a circle?

**Ringing:** How much negative resistivity (that makes no physical sense given a positive ground truth) is reconstructed?

Obviously, not only the figures of merit at a single location, but also their change
from the center to the boundary is of interest, since this gives a measure of the homogeneity of the algorithm under test.

Second, GREIT comes with a method of reconstruction. One way of interpreting it is to see it as an algorithm that is learned using pairs of training data. Here, these pairs consist of known resistivity distributions $Z^{GT}$ with point-shaped resistivity disturbances of the background and their corresponding voltage vectors $u^{GT}$ obtained via FEM simulation. Additionally, pairs of voltage vectors resembling noise and their desired reconstructed resistivity distributions (which are zero) are included. Using those, a linear relationship $\tilde{G}$ that minimizes a quadratic cost function can be learned, i.e.

$$\tilde{G} = \arg\min_{G} ||Z^{GT} - G \cdot u^{GT}||_2.$$  

(3)

This is described in more detail in section 3.3.

It should be noted that in GREIT, conductivity- instead of resistivity-distributions are used. In lung EIT, however, inhalation, i.e. an increase in air in the lungs is generally associated with a positive signal. Hence, resistivity-distributions are used in this paper.

The evaluation and reconstruction based on point-shaped resistivity disturbances presented in GREIT are included in EIDORS and have found widespread acceptance in the community. At the same time, however, point-shaped resistivity distributions are an unrealistic assumption in certain applications like, for example, pulmonary monitoring. We thus ask the question if a GREIT-type method of evaluation and reconstruction can be developed that operates with datasets that are more tailored towards resistivity distributions found in those real-world applications. By including this type of a-priori knowledge, we expect to see an improvement of image quality in terms of contrast and reduction of artifacts. At the same time, objective comparison and numeric evaluation of reconstruction results must be maintained.

To achieve this, novel figures of merit are presented in the next section that operate with spatially extended shapes but are based on the ones introduced in GREIT. Moreover, a large database of evaluation shapes is created from publicly available CT data. Two GREIT-type reconstruction algorithms are developed using orthonormal training data sets obtained using Singular Value Decomposition (SVD) from two different sources of raw data and a tunable desired point-spread function. In section 4, the novel methods of reconstruction are compared against the standard point-based GREIT approach using the novel figures of merit. In the next section, visual comparison of reconstruction results of real EIT data is performed. Finally, section 5 concludes the paper. Preliminary findings related to this article have been previously published at the EIT conference 2013 (Hoog Antink et al., 2013).

3. Materials & Method

3.1. Novel Figures of Merit

As mentioned above, the novel figures of merit (FOM) are able to process spatially extended evaluation data but are based on the ones defined in GREIT. Table 1 lists the
Table 1. Novel Figures of Merit

<table>
<thead>
<tr>
<th>Novel Figure of Merit</th>
<th>GREIT Equivalent</th>
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<tbody>
<tr>
<td>Widening (WD)</td>
<td>Resolution</td>
</tr>
<tr>
<td>Overshoot (OS)</td>
<td>Ringing</td>
</tr>
<tr>
<td>Distortion (DT)</td>
<td>Shape Deformation</td>
</tr>
<tr>
<td>Dislocation (DL)</td>
<td>Position Error</td>
</tr>
<tr>
<td>Impedance/Area Ratio (IAR)</td>
<td>Amplitude Response</td>
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</table>

Figure 1. Illustration of the definitions used in the novel FOMs.

In GREIT, the properties of the reconstruction are assumed to be cylindrical symmetric. Thus, the evaluation can be described exhaustively by the distance from the center. In our approach, we remove the constraint of cylindrical symmetry and expand the evaluation data to a set of \(i\) arbitrary shapes which are represented by binary images \(B_{i}^{GT}(x, y)\) where 1 denotes shape and 0 background, see Figure 1 I. It thus has the corresponding area

\[
A_{i}^{GT} = \sum_{x,y} B_{i}^{GT}(x, y). \tag{4}
\]

As medical image processing is generally performed on a rectangular pixel grid, let us assume such a grid with \(x, y \in [1, 2, ..., 32]\). As in GREIT, from these ground truth resistivity distributions, voltage vectors are calculated using the FEM solver included in EIDORS. Those are passed to the algorithm under test. As its output, we define...
$Z_i(x,y)$ to be the reconstructed resistivity image, Figure 1 II. $\hat{Z}_i$ is the amplitude, i.e. the maximum value of $Z_i(x,y)$. The distribution of negative resistivity $Z_i^{<0}$ is defined as

$$Z_i^{<0}(x,y) = \begin{cases} Z_i(x,y) & \text{for } Z_i(x,y) < 0 \\ 0 & \text{otherwise.} \end{cases}$$

For the quarter amplitude set (QAS), the same definition as in GREIT is used. It is thus a binary image with

$$B_i^{QAS}(x,y) = \begin{cases} 1 & \text{for } Z_i(x,y) > 0.25\hat{Z}_i \\ 0 & \text{otherwise.} \end{cases}$$

The area of the QAS is defined as $A_i^{QAS}$ in analogy to the area of the ground truth, see Figure 1 III. With these definitions, the performance metrics are defined as follows.

**Widening** The FOM Widening (WD) is defined similarly to Resolution: It is the ratio of area of the QAS to the area of the evaluation shape, i.e.

$$WD_i = \frac{A_i^{QAS}}{A_i^{GT}}.$$  

The larger WD, the blurrier the reconstructed resistivity image. For the whole evaluation data set, the mean value is calculated,

$$\overline{WD} = \text{mean}(WD_i).$$

**Overshoot** Analog to Ringing, the FOM Overshoot (OS) is the ratio of the sum of reconstructed negative resistivity to the sum of absolute resistivity,

$$OS_i = \frac{-\sum_{x,y} Z_i^{<0}(x,y)}{\sum_{x,y} |Z_i(x,y)|}. $$

The larger OS, the more ringing artifacts are present. Here, the mean value is calculated as well,

$$\overline{OS} = \text{mean}(OS_i).$$

Note that the definition of Overshoot is essential the same as Ringing. It is introduced to clearly distinct between point- and shape-based evaluation.

**Distortion** The definition of Distortion (DT) is

$$DT_i = \frac{\sum_{x,y} |B_i^{QAS}(x,y) - B_i^{GT}(x,y)|}{A_i^{GT}}.$$ 

It is thus similar to Shape Deformation in classical GREIT. At the same time, it shows a strong similarity to WD and will produce only significantly different results if the
ground truth is not completely enclosed by the QAS, see Figure 1 IV. Thus, the larger the DT (relative to WD), the more distorted the image. Here, the mean value over the whole evaluation data set is of interest,

$$\overline{DT} = \text{mean}(DT_i).$$

**Dislocation** The point based evaluation allows a signed *Position Error*. Since it is possible in our approach that the result of the reconstruction is shifted in two dimensions, the *Dislocation* (DL) is defined as an unsigned distance ratio

$$DL_i = \frac{\sqrt{\|\vec{C}_{GT_i} - \vec{C}_{QAS_i}\|^2}}{R_T},$$

(13)

where $R_T$ denotes the radius of the complete image and $\vec{C}_{GT, QAS}$ denotes the center of gravity of the ground truth or the reconstructed resistivity distribution, respectively (see Figure 1 V). Thus, a larger DL indicates a greater error in the position of the reconstructed test shape. Here, the mean value is calculated as well

$$\overline{DL} = \text{mean}(DL_i).$$

**Impedance/Area Ratio** The *Amplitude Response* is a very important parameter in GREIT: If it is constant, it allows exact quantitative analysis of, for example, air distribution inside the lung. The novel *Impedance/Area Ratio* (IAR) has a similar purpose and is defined as the ratio of the sum of the reconstructed resistivity and the area of the evaluation shape,

$$\text{IAR}_i = \sum_{x,y} Z(x, y) \frac{A_{GT}}{A_{GT}}.$$

(15)

If IAR was constant for a whole set of $i$ evaluation shapes, it could be compensated with a simple factor. If it varies, however, compensation is impossible. We therefore define IAR on the whole set by

$$\text{IAR}^* = \frac{\text{std}(\text{IAR}_i)}{\text{mean}(\text{IAR}_i)},$$

(16)

for a set of $i > 2$ evaluation shapes with std($x$) being the standard deviation. The larger IAR, the more difficult it is to determine the size of an resistivity distribution from the global impedance.

### 3.2. Evaluation Shapes

The novel figures of merit can be used on any type of reconstruction with any type of shape. For a specific application, the selection of expected shapes seems reasonable. For the chosen application of pulmonary imaging, a publicly available CT database (www.dir-lab.com (Castillo et al., 2009)) was used. It consists of 4D-CT images, where
Figure 2. Example of 40 lung shapes sorted by area. The lung shapes are displayed in white against the circular black domain indicating the homogeneous background.

Figure 3. The flat slope of the normalized cumulative sum of the singular values of the proposed evaluation data set indicates little linear dependence.

The breathing cycle is subdivided into ten phases $T = 00\%, T = 10\%, \ldots, T = 90\%$. Images of a total of ten patients that underwent radiation treatment for lung cancer are available. For every patient, the first six phases $T = 00\%$ (end of inspiration) to $T = 50\%$ (end of expiration) were selected, resulting in 6930 traversal images. An algorithm was developed that automatically segments the shape of the left and right lung or discharges the image. This resulted in a total of 2759 pairs of lung shapes. A selection of 40 lung shapes sorted by area can be found in Figure 2. The great variation in size originated from the different phases of the breathing cycle and (most important) from the different position of the CT slice along the craniocaudal axis. Moreover, they also originate from the different patient boundary shapes. Singular value analysis was used to determine the richness of the evaluation data, see Figure 3. It shows that the first 104 singular values contained only a little more than 60% of the total sum and the first 350 singular values contained 90% of the total sum of singular values. If the dataset was extremely biased, for example contain only one image of a left and one image of a right lung, only the first two eigenvalues would be greater than zero.

For better comparability to the original GREIT algorithm, the proposed reconstructions algorithms were additionally evaluated with point-shaped test data.
Using those, the novel figures of merit are almost identical to those implemented in GREIT.

### 3.3. Reconstruction Algorithm

As argued above, GREIT can be interpreted as a learning algorithm. Let us denote $u$ to be the vector of voltages measured on the patients surface. We seek to reconstruct the resistivity distribution $Z$. In a linear case, this can be achieved by matrix multiplication, i.e.

$$Z = Mu.$$  \hspace{1cm} (17)

If we have a set of $N$ known resistivity images $Z_i'$ and the corresponding voltage distributions $u_i'$, we can learn the matrix $M$ that minimizes some cost function between desired output $Z_i'$ and result of the reconstruction $Z_i$. In EIDORS, this norm was selected to be the $l2$-norm, which allows the calculation of $M$ via matrix inversion. As in GREIT, virtual noise measurements $u_i''$ to represent electronic measurement noise and the corresponding desired outputs

$$Z_i'' = 0.$$  \hspace{1cm} (18)

are introduced. In matrix notation, the combination of desired output / input combinations and virtual noise measurements can be expressed as

$$Z' = Mu',$$  \hspace{1cm} (19)

with

$$Z' = [Z_1', Z_2', ..., Z_N', 0, 0, ..., 0]$$  \hspace{1cm} (20)

and

$$u' = [u_1', u_2', ..., u_N', u_1'', u_2'', ..., u_M''],$$(21)

where $M$ is the dimension of the voltage vector. In a 16 electrode system with the Sheffield measurement pattern, this value is

$$M = 16 \cdot 13 = 208.$$  \hspace{1cm} (22)

This allows the calculation of the reconstruction matrix via

$$M = Z'u'^T(u'u'^T)^{-1}.$$  \hspace{1cm} (23)

In GREIT, the training data $Z_i'$ consists of blurred point-sources and zero vectors. Choosing the appropriate coordinate system, these point sources can be expressed as

$$Z_1' = [1, 0, ..., 0], Z_2' = [0, 1, ..., 0], ..., Z_N' = [0, 0, ..., 1].$$  \hspace{1cm} (24)
We can see that this forms an orthonormal representation of the resistivity space, i.e.

\[ Z'_i \cdot Z'_j = \delta_{ij}, \]  

with \( \delta_{ij} \) being the Kronecker Delta.

If reconstruction was a linear problem, the choice of training data could be an arbitrary one, as long as the whole resistivity-space is spanned. However, since reconstruction is a nonlinear, inverse, ill-posed problem, the training data will have an influence on the learned reconstruction matrix. As argued above, point-shaped resistivity distributions may not be a realistic scenario for certain applications. We thus want to examine the possibility of using spatially extended data to train a GREIT-type reconstruction algorithm. At the same time, to maintain a neutral set of training data, the condition of orthonormality should be maintained. This can be achieved by using the Singular Value Decomposition (SVD). Let \( \mathbf{R} \) be the candidate training data set in Matrix notation, consisting of \( P \) resistivity images \( \tilde{Z}_i \), i.e.

\[ \mathbf{R} = \begin{bmatrix} \tilde{Z}_1, \tilde{Z}_2, \ldots, \tilde{Z}_P \end{bmatrix}. \]  

(26)

Via SVD, this matrix can be decomposed into three matrices,

\[ \mathbf{R} = \mathbf{U} \mathbf{S} \mathbf{V}^*. \]  

(27)

The matrix \( \mathbf{U} \) contains the singular vectors,

\[ \mathbf{U} = \begin{bmatrix} U_1, U_2, \ldots, U_P \end{bmatrix}, \]  

(28)

which can be interpreted as “eigenimages” that satisfy the condition of orthonormality. In a 16-electrode scenario, the voltage vectors contain 208 values, from which only 104 are linearly independent. It is thus sufficient to select only those eigenimages belonging to the 104 greatest singular values, i.e.

\[ \tilde{Z}'_i = U_i \quad \text{for } i = [1, 2, \ldots, 104] \]  

(29)

as training data.

As in GREIT, the training resistivity images \( \tilde{Z}'_i \) are used in a forward simulator to generate the corresponding voltage vectors \( u'_i \). To generate the desired resistivity images \( Z'_i \), an additional step is introduced. Let us define ideal reconstructed EIT to be a spatial low-pass filter of the truth, in analogy to an optical system that will introduce some sort of blur but no further distortion. We can thus describe it as a convolution with some filter kernel \( H_{LP} \). Here, we use a Gaussian spatial low-pass filter with variance \( \sigma \), defined on a rectangular pixel grid:

\[ Z'_i = \tilde{Z}'_i * H_{LP}(\sigma), \]  

(30)

where \( * \) denotes the spatial convolution operator.
In GREIT, the handling of noise plays an important role. The concept of a noise figure $\eta$ is introduced. It describes the ratio of signal to noise in the reconstructed image to the signal to noise in the voltage data,

$$\eta = \frac{\text{SNR}(Z)}{\text{SNR}(u)}. \quad (31)$$

As in GREIT, the amplitude of $u''_i$ is optimized iteratively until a suitable noise figure of, in this case $\eta = 0.5$, is reached. This is similar to the optimization of the hyperparameter $\lambda$ in regularization-based reconstruction methods. An outline of the described algorithm is found in Figure 4.

### 3.4. Candidate Training Data

Two different sets of candidate training data are used to learn the SVD GREIT reconstruction matrix. The first set consists of 1000 lung shapes randomly drawn from the 5518 that are used in the evaluation process. From this small subset of the evaluation data, 104 eigenimages are used for training while the evaluation set consists of the complete 5518 resistivity images. The resulting algorithm is termed “GREIT SVD GT”.

The second set of candidate training data is completely independent from the evaluation data. In an animal trial, EIT was recorded on an mechanically ventilated but otherwise healthy pig using the Dräger EEK2 device. 4800 frames were reconstructed using the manufacturer’s proprietary method of reconstruction, see Figure 5. As above, only the first 104 eigenimages (and no real voltage measurements) are used to train the algorithm termed “GREIT SVD RD”.

![Figure 5. Ten reconstructed EIT frames using the manufacturers proprietary method of reconstruction. The time difference between frames is 500ms.](image)

The first 10 eigenimages for both scenarios are shown in Figure 6.
Figure 6. The first ten eigenimages of the subset of 1000 lung shapes (top row, used to train GREIT SVD GT) and of the reconstructed EIT frames (bottom row, used to train GREIT SVD RD). Note that images in the top row show much sharper edges.

Table 2. Algorithm performance evaluation for $\sigma = 2$, $\eta = 0.5$ (smaller is better). The optimal performance for each FOM is highlighted.

<table>
<thead>
<tr>
<th>FOM</th>
<th>Lung Shape Evaluation</th>
<th>Point Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GREIT PT SVD GT SVD RD</td>
<td>GREIT PT SVD RD</td>
</tr>
<tr>
<td>WD</td>
<td>1.8589 1.4225 1.7961</td>
<td>79.7845 73.7251 79.7279</td>
</tr>
<tr>
<td>OS</td>
<td>0.0038 0.0026 0.0063</td>
<td>0.0201 0.0836 0.0241</td>
</tr>
<tr>
<td>DT</td>
<td>1.8595 1.4266 1.7970</td>
<td>79.7845 73.8522 79.7279</td>
</tr>
<tr>
<td>DL</td>
<td>0.0257 0.0192 0.0246</td>
<td>0.0237 0.0445 0.0236</td>
</tr>
<tr>
<td>IAR$^*$</td>
<td>0.1311 0.1411 0.1171</td>
<td>0.4535 0.5828 0.4572</td>
</tr>
</tbody>
</table>

3.5. Reference Algorithm

As a method of reference, a classical point-based GREIT algorithm was implemented and termed “GREIT PT”. Here, a training dataset of point-shaped resistivity distributions was created on the 32 by 32 pixel grid. Since only points inside a circular area were considered, a set of $N = 748$ resistivity distributions $\tilde{Z}'$ was used. These images were obviously not processed via SVD, the rest of the algorithm (spatial low-pass filtering, forward simulation, etc.) remained unchanged.

4. Results & Discussion

The results for the different reconstruction matrices obtained with a fixed $\sigma = 2$ are presented in Table 2.
Using the novel figures of merit and evaluation data, an increase in performance over the point-based approach can be made out in all FOMs except IAR using the GREIT SVD GT approach. On the other hand, when using GREIT SVD RD, a smaller but still existing improvement in all FOMs except OS was observed. This is significant since training and evaluation data come from two completely independent sources in this scenario.

If, for comparison, point-shaped test data is used, the GREIT PT algorithm shows superior performance over GREIT SVD GT in the majority of FOMs. As with the lung shape evaluation, GREIT SVD RD shows very similar behavior to GREIT PT and GREIT SVD GT outperforms both in terms of WD and DT.

In Figure 7 the results for a variation of \( \sigma \in (1, 4) \) is shown. A further decrease in \( \sigma \) resulted in severe image distortion, please see also Figure 8. Several observations can be made. First, a dependence of the performance of the reconstruction on \( \sigma \) can be observed. It can be inferred from the OS plot that the amount of ringing artifacts can be reduced by increasing \( \sigma \). At the same time, an increase in WD can be observed.
Figure 8. Visual comparison of the different algorithms and variations in $\sigma$. The last row shows a difference image. Note that GREIT SVD GT produces a shape most similar to the ground truth (Figure 9). Further note that for $\sigma = 0$ severe image distortion occurs.

Figure 9. Ground truth sample shape used for Figure 8.

This is most prominent for the GREIT SVD GT approach. In general, its superior performance in all FOMs except IAR can be observed for $\sigma \in (1, 3)$. For $\sigma > 3$ it shows a strong deterioration in all FOMs except OS. The GREIT SVD RD approach shows the best performance in IAR over all values of $\sigma$. For OS, however, it shows the worst performance. Comparing the plots for DT and WD, one cannot observe any difference. This proves that for the great majority of evaluation shapes, the ground truth is completely enclosed by the reconstructed shape.

In Figure 8, a visual comparison of the different algorithms for varying values of $\sigma$ when reconstructing an evaluation shape (Figure 9) is given. All resistivity images were normalized so that $\hat{Z}_i = 1$. It confirms that differences between GREIT PT and GREIT
SVD RD are rather small. It also shows that the variation of $\sigma$ plays an important role. The GREIT SVD GT shows a slightly different behavior that is most obvious in the difference image (bottom row). Comparing the results to the ground truth shape given in Figure 9, one can see that GREIT SVD GT with $\sigma = 1$ best captures the concave nature of the shape.

Examining Figure 6, a hypothesis can be formulated: While the eigenimages used to train GREIT SVD RD show a smooth behavior, the eigenimages used to train GREIT SVD GT show sharp jumps and small structures. This could explain the varying performance for the different figures of merit. It would also explain why the performance deteriorates greatly for rising $\sigma$, since small structures might be blurred completely. This would result in very similar desired resistivity outputs $Z'$ for different voltage inputs $u'$.

To evaluate the algorithm with real EIT data, voltage recordings from the animal trial described in section 3.4 were reconstructed using the three algorithms with $\sigma = 1, 2, 3, 4$. Using a PCA-based separation approach similar to the one described by (Deibele et al., 2008), a respiration- and a perfusion-related image were extracted. The results are shown in Figure 10. Here, the differences between GREIT PT and GREIT SVD RD are also very small. GREIT SVD GT however shows a different behavior that is most prominent for $\sigma = 1$: While the other algorithms show a (physiological unreasonable) respiratory component very close to the dorsal boundary, no such artifacts can be seen in the results produced by GREIT SVD GT. In general, the lung contours produced with this algorithm show a much clearer definition, which is consistent with the observation that it showed the best performance in terms of widening. The same increase in contrast can be observed in the perfusion-related image. Here, a clearly defined heart-region as well as two lung-regions can be observed. Moreover, heart- and lung-region show a significant phase-shift, which also agrees with physiology. As expected from analysis using the novel FOMs (Figure 7), these properties deteriorate for $\sigma \geq 3$.

5. Conclusion

The first conclusion that can be drawn is that using spatially extended, orthonormal training data obtained via SVD to train a GREIT-type reconstruction algorithm is indeed feasible. Depending on the choice of training data, it can lead to similar reconstruction results compared to the standard approach and can even increase performance in certain areas, for example better image contrast. Moreover, the introduction of a novel desired point-spread function with tunable blurring $\sigma$ introduces another degree of freedom for tuning the reconstruction towards a specific goal.

Second, the proposed figures of merit in combination with the large data set of realistic lung-shapes provides a powerful method of evaluation. Comparing the results from this analysis to real EIT reconstructions, good agreement can be observed. For example, a numerical decrease in widening resulted in a visually observable increase in contrast.
The incorporation of anatomical information into the EIT reconstruction problem has a comparatively long history (Vauhkonen et al., 1997). The proposed approach has the advantage that it presents a framework for the automated synthesis and analysis of EIT reconstruction algorithms based on realistic lung shapes automatically segmented from CT data. Moreover, it allows the off-line calculation of a reconstruction matrix which can be used for linear reconstruction.

There is no optimal method of linear reconstruction per se - depending on the intended application, tuning of the reconstruction is possible and may optimize some meta-level goal. The regular GREIT-evaluation has the clear advantage of using very simple evaluation data that can be used for any type of application. We thus consider our approach as an extension that allows tailoring towards a specific case. An important question remaining is, which figures of merit need to be optimized for which type of application. A recent study indicates that certain parameters in thoracic EIT show no significant change even between GREIT or the supposedly outdated back projection algorithm (Zhao et al., 2013). It can be assumed, however, that this will not be the
case for every analysis. We suspect that if, for example, local perfusion is examined, the fine spatial resolution needed and the smaller amplitude of the signal will call for an optimized reconstruction. Here, the proposed GREIT SVD GT can serve as a starting point. At the same time, this is an area that needs to be approached with caution, since one of the intentions of the GREIT concept is to increase comparability of EIT results. The novel figures of merit are designed with the same goal in mind. To fully achieve it, we propose to develop a standardized set of evaluation shapes and distribute it with the open-source EIDORS package. Moreover, an extension of the method to three-dimensional data is desirable. While this would require large amounts of three-dimensional, segmented CT data, the translation of the general concept should be straightforward.

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